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# Middlemen and the Allocation of Heterogeneous Goods

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**Abstract**

This paper presents a general equilibrium model in which middlemen emerge to facilitate trade in an environment of idiosyncratic tastes and heterogeneous goods. The gains to the traders can be measured along three dimensions: the rate of production, the time preference losses generated by the matching process, and the quality of the match between consumers' preferences and the goods they ultimately consume.

## 1. Introduction

A middleman acts as an intermediary between the seller of a good and its potential buyers. Agents adopt this role because it is at least as profitable as the other roles open to them, but what service do middlemen provide that allows them to earn these profits? And how does their presence alter the efficiency of the economy? These issues are central to an understanding of middlemen. In this paper, we study the role of middlemen in an economy in which goods are heterogeneous and tastes are idiosyncratic. Ideally, each unit of goods should be allocated to an agent who places a very high value on it, but a less desirable allocation is attained when trading frictions are present. We argue that middlemen ameliorate these

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frictions by holding inventories. Consumers who meet these middlemen are more likely to obtain a good that suits their tastes.

Rubinstein and Wolinsky [5] and Yavas [7, 8] argue that the role of middlemen is to reduce the time preference losses that occur when agents must search for trading partners, but their explanation is not entirely satisfactory. Rubinstein and Wolinsky [5], for example, imagine an economy in which there are three groups of people: sellers, buyers, and middlemen. The buyers want to consume units of a homogeneous good that are initially held by the sellers. The middlemen do not possess or consume units of the good. The agents are randomly and repeatedly matched, and transactions can occur only between matched agents. Rubinstein and Wolinsky show that there can be an equilibrium in which middlemen buy goods from sellers and re-sell them to buyers.[1] However, if middlemen have no inherent advantage over the other agents—if they are neither more patient nor more easily matched—each group of agents is just as well off when the middlemen are active as they are when the middlemen are idle. Furthermore, *any* inherent advantage causes middlemen to participate in the trading, because middlemen are not given the option to assume the role of buyer or seller.

An alternative explanation of the role of middlemen is based upon the moral hazard problem that arises when product quality is not immediately observable. Biglaiser [1] and Biglasier and Friedman [2] describe environments in which the quality of the middleman's goods is more predictable than the quality of the producer's goods. Since customers prefer to deal with a middleman, middlemen can purchase goods in large quantities. The size of their purchases allows them to employ quality control methods that are unavailable, or unprofitable, to individual consumers, and these controls ensure that their goods continue to be of predictable quality.[2]

Li [4] presents a general equilibrium version of Biglaiser's model. Li emphasizes

[1] A seller who is matched with a middleman must choose between selling to the middleman and waiting for a match with a buyer. He has a positive rate of time preference, so he will sell his unit of goods to the middleman for less than he would have sold it to a buyer. Similarly, a buyer who is matched with a middleman must choose between buying from the middleman and waiting for a match with a seller. He also has a positive rate of time preference, so he will buy from the middleman at a somewhat higher price than he would have been willing to buy from a seller. The middleman's undiscounted profits are equal to the gap between his sale and purchase prices.

[2] Biglaiser argues that middlemen use quality controls that would be too costly for individual consumers. Biglaiser and Friedman argue that middlemen obtain goods from several competing sources. The middleman can then enforce quality standards by threatening to drop from its list of suppliers any firm that is found to be providing low quality goods.

the moral hazard problem generated by the producer's ability to produce low quality goods that cannot be immediately identified as such by the consumer. Middlemen emerge by acquiring, at some cost to themselves, the ability to verify the quality of goods. They are able to sell goods at a higher price than producers because consumers believe that the middlemen are more likely to sell high quality goods. Li's middlemen are expert traders whose "real world" counterparts are found in the markets for used cars, computers, art works, and many other goods.

This paper proposes a different role for middlemen. It is assumed that goods are heterogeneous and that consumers have idiosyncratic tastes, so that the way in which goods are allocated to consumers matters. Middlemen hold inventories, so that they are better able to match people with goods. The "real world" counterparts of these middlemen are found in the markets for housing, new and used cars, clothing, and many forms of retail trade.

Furthermore, it is assumed that each supplier of goods can freely choose between the roles of middleman and producer. If improvements in the manner in which goods are allocated to consumers are of little value (as sometimes occurs), every supplier will choose to be a producer, and mediation will not occur. Under other circumstances, the quality of the match between consumers and goods is sufficiently important that some suppliers will choose to be middlemen rather than producers.

An important aspect of this approach is that the efficiency of trade has three dimensions: the time preference losses induced by the delay in completing transactions, the quality of the match, and the rate of production. We find that middlemen influence each of these dimensions:

(a) An equilibrium with middlemen always has higher match quality than an equilibrium without them, partly because middlemen offer consumers a wider variety of goods and partly because middlemen charge a higher price (so that consumers require a better match before they will buy).

(b) There is some tendency for the rate of production to be higher in the presence of middlemen. In equilibrium, the rate of production is equal to the rate of consumption. Mediation will result in faster production if middlemen sell goods to consumers more quickly than do producers. Middlemen offer a greater variety of goods than do other sellers, and hence are more likely to conclude a sale once they have been matched. However, middlemen also charge higher prices, making them less likely to conclude a sale. The former effect tends to dominate the latter, so that the rate of production tends to be higher when middlemen are present.

(c) The efficiency of the trading process depends in part upon the delay between the production of a unit of goods and its ultimate consumption. (Indeed, this is the only aspect of efficiency described by the Rubinstein and Wolinsky model.) In our model, these delays are proportional to the quantity of goods held in the economy's inventories. Since middlemen hold larger inventories than other suppliers, the presence of middlemen raises the economy's inventories and increases the trading delays.

The welfare effects of the first two factors tend to outweigh the welfare effect of the last, so that welfare tends to be higher in the presence of middlemen.

Shevchenko [6] describes a model of mediation which, like ours, is based on heterogeneous tastes. He imagines an economy in which agents are both producers and consumers, and in which goods of a finite number of types are produced and consumed. Agents are forced to search because they produce one type of good but wish to consume another. Trade between such agents requires a double coincidence of wants: each agent must have produced the good that the other agent wants to consume. Middlemen facilitate trade by holding an inventory of goods. If an agent meets a middleman, and if that middleman possesses the good that he wishes to consume, the agent swaps the good that he produced for the good that he wants to consume. The middleman collects a commission for this service. The commission is determined by the Nash bargaining solution, and varies with the composition of the middleman's inventory. It is largest when the middleman has few units of the good consumed by the agent and many units of the good produced by the agent.

Although they have a number of common features, Shevchenko's paper and our paper focus on different issues. In Shevchenko's model, a good is either acceptable to a consumer or it is not. In our model, every unit is acceptable to every agent at some price. The surplus generated within an economy depends not only on how quickly goods are produced and sold, but also on the way in which they are allocated to the agents. The quality of the match between goods and agents is endogenous variable, and is one of the major determinants of economic welfare.

The difference in the role played by heterogeneity gives rise to a difference in the manner in which prices are set. In Shevchenko's model, each transaction alters the composition of the middleman's inventory but not its size. The price that the middleman extracts for his services depends upon the way in which the transaction changes the composition of his inventory. By contrast, in our model, each sale out of inventory reduces the size of the inventory, and restoring the size of the inventory involves the middleman in costly search. The middleman sets his price by trading off the benefit of a sale and the cost of search.

Finally, the papers treat inventory size and the entry decision differently. Our model restricts middleman to only two units of inventory, and imagines that a prospective middleman must bear the costs of acquiring his inventory. Specifically, he must forgo the sale of his first unit, search for someone to sell him a second unit, and then pay the market price for it. By contrast, Shevchenko's model determines endogenously the optimal size of the middleman's inventory. The entry decision is, however, somewhat simplified: each agent chooses between being a producer-consumer, or a middleman possessing a fully stocked inventory. The origin of these inventories is not described.

## 2. A Model of Middlemen

Consider a market in which units of a divisible **numeraire good** are exchanged for units of an indivisible **heterogeneous good**. These goods have the following properties:

(a) The numeraire good's consumption value is equal to its production cost, and is normalized at unity.

(b) The consumption value of the heterogeneous good varies across units for each buyer and across buyers for each unit. These valuations are known to the buyer but not observed by sellers. They are modelled as independent (across both units and buyers) realizations of the random variable $\theta$, which has density $f(\theta)$ and cumulative distribution $F(\theta)$ defined on the unit interval. The hazard function $h(\theta) \equiv f(\theta)/[1-F(\theta)]$ is assumed to be increasing and once differentiable. The cost of producing a unit of the heterogeneous good is $\gamma < 1$.

Both goods are produced instantly as needed.

The market consists of a fixed number of agents. Every agent is infinitely lived and risk neutral, and every agent is able to produce the numeraire good. A fraction $1-N$ of the agents are **consumers** of the heterogeneous good while the fraction $N$ of the agents supply it. Suppliers have two attributes. First, each supplier can hold up to two units of the heterogeneous good in his inventory.

Second, each supplier is able to produce a unit of the heterogeneous good (at cost $\gamma$) if he is not currently holding a unit of that good. The assumption that a supplier who is holding a unit of goods cannot produce restricts the behaviour of suppliers in two important ways. A supplier can only acquire a two-unit inventory by buying the second unit from another supplier. A supplier must find a buyer for each unit of goods that he produces before he can produce another.

Suppliers who are currently holding one unit of the heterogeneous good are called **producers**, because they can maintain their one-unit inventories by producing a new unit after each sale. Suppliers who are currently holding two units of goods are called **middlemen**. Suppliers will typically switch between these roles as time passes: the sale of a unit of goods by a middleman converts him into a producer, and the purchase of a unit of goods by a producer converts him into a middleman. However, at any given moment,

$$N = P + M \tag{2.1}$$

where $P$ and $M$ are the fractions of agents who are producers and middlemen respectively.

The agents are randomly and repeatedly matched such that any agent will meet another agent, within some arbitrarily short interval, with probability $\alpha$. Some of these meetings result in trades in which units of the numeraire good are swapped for a single unit of the heterogeneous good.

(a) A producer who meets a consumer offers to sell his unit of the heterogeneous good at the price $p_C$. The consumer observes the type of the good, and if he decides to purchase it, produces the required $p_C$ units of the numeraire good. The goods are then exchanged and consumed, and the producer produces a new unit of the heterogeneous good.

(b) A meeting between two producers might result in a trade. One of the producers could become a middleman by purchasing the other's heterogeneous good. The other producer would consume the numeraire goods received in payment, produce another unit of the heterogeneous good, and continue as a producer. This transaction occurs if the value of becoming a middleman is sufficiently high, and not otherwise. The price $p_M$ at which the heterogeneous good is traded, if it is traded, leaves the two producers indifferent as to which side of the transaction they take.

(c) A middleman will, upon meeting a consumer, offer the consumer his choice of the goods at the price $q$. The consumer buys the better of the goods or neither of them. If a sale occurs, the middleman consumes the numeraire goods received in payment, and becomes a producer.

### 2.1. Consumers

Let $W$ be the expected present discounted utility (EPDU) of a consumer. A consumer will, over an arbitrarily short interval, meet a producer with probability $\alpha P$ and meet a middleman with probability $\alpha M$. The consumer will buy a producer's unit of the heterogeneous good if his valuation of that good exceeds its price $p_C$. The consumer will buy the better of two units from a middleman if his valuation of that unit exceeds its price $q$.[3] Thus, the flow utility of a consumer is:

$$rW = \alpha P \int_{p_C}^{1} (\theta - p_C) f(\theta) d\theta + \alpha M \int_{q}^{1} (\theta - q) \left[2f(\theta)F(\theta)\right] d\theta \qquad (2.2)$$

where $r$ is the discount rate. The first integral is the expected benefit of a meeting with a producer, and the second integral is the expected benefit of a meeting with a middleman.[4]

### 2.2. Producers

Producers sell units of the heterogeneous good to consumers and to other producers, and they purchase units of goods from other producers (thereby becoming middlemen).

When a producer meets a consumer, the value that the consumer places on the unit that he possesses is not known to the producer. The producer sets $p_C$ to maximize the expected profits from each meeting with a consumer.

When two producers meet, and one producer sells a unit of goods to the other, each producer's welfare is altered in a different way. The seller of the good receives

[3] Middlemen would be more likely to be viable if a consumer were permitted to purchase both units of a middleman's inventories. The contrary assumption, that the consumer purchases at most one unit, was chosen to show that this advantage is not necessary for the success of middlemen.

[4] Here, $f(\theta)F(\theta)$ is the density function of the event that a particular good in a pair is the better of the two and has value $\theta$ to the consumer. The density function of the event that the better good has value $\theta$ to the consumer, is twice this density, because each good is equally likely to be the better of the two.

a payment $p_M$, replaces the unit at cost $\gamma$, and continues as a producer. The benefit that he receives from the transaction is simply $p_M - \gamma$. The purchaser of the good incurs a cost $p_M$, and his status changes from producer to middleman. If $V$ and $U$ are the expected present discounted utilities of the producer and middleman respectively, the benefit that he receives from the transaction is $U - V - p_M$. If these benefits were not equal, both producers would want to be on the same side of the transaction—both would want to be the seller or both would want to be the buyer. The price $p_M$ is assumed to adjust to equalize the benefits, leaving the producers indifferent as to which side of the transaction they take. That is, the price at which producers trade satisfies the condition:

$$p_M - \gamma = U - V - p_M \tag{2.3}$$

implying

$$p_M = (\Delta + \gamma)/2$$

where:

$$\Delta \equiv U - V$$

When $p_C$ and $p_M$ are set in this fashion, the flow benefit to being a producer is:

$$rV = \alpha C \max_{p_C} (p_C - \gamma)(1 - F(p_C)) + \alpha P \max[0, p_M - \gamma]$$

The first max function is the expected profits from a meeting with a consumer, and the second max function is the expected profits from a meeting with a producer.

Define the functions:

$$\eta_x(z) \equiv \max_p \{(p - z)(1 - F(p)^x)\} \qquad x \in \{1, 2\}, z \in [0, 1]$$

$$\pi_x(z) \equiv \arg\max_p \{(p - z)(1 - F(p)^x)\}$$

The restrictions on the hazard function imply that $\pi_1$ and $\pi_2$ are increasing functions, and that $\eta_1$ and $\eta_2$ are decreasing and convex functions. Using this notation, and using (2.3) to eliminate $p_M$, yields:

$$rV = \alpha C \eta_1(\gamma) + \alpha P \max[0, (\Delta - \gamma)/2] \tag{2.4}$$

$$p_C = \pi_1(\gamma) \tag{2.5}$$

## 2.3. Middlemen

A middleman meets a consumer with probability $\alpha C$. This meeting might result in a sale of one unit of goods at price $q$, and a switch in status from middleman to producer. Thus, the flow benefit to being a middleman is:

$$rU = \alpha C \max_q (q - \Delta)[1 - F(q)^2] = \alpha C \eta_2(\Delta) \tag{2.6}$$

The price $q$ at which middlemen sell to consumers is:

$$q = \pi_2(\Delta) \tag{2.7}$$

## 2.4. Entry and Exit

As time passes, middlemen sell goods to consumers, thereby becoming producers, and producers buys goods from other producers, thereby becoming middlemen. The number of middlemen and the number of producers are constant through time if middlemen are becoming producers with the same frequency as producers are becoming middlemen. This condition can be written as:

$$(\alpha P^2/2) I(\Delta - \gamma) = M\alpha(1 - N)(1 - F(q)^2) \tag{2.8}$$

where the indicator function $I$ is equal to 1 if producers trade with each other and equal to 0 if they do not:[5]

$$I(\Delta - \gamma) = \begin{cases} 1 & \text{if } \Delta - \gamma \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The left-hand of (2.8) is the rate at which producers are becoming middlemen. Over an arbitrarily short interval, any given producer meets another producer with probability $\alpha P$. If trade occurs between them, that producer becomes a middleman with probability 1/2 (and remains a producer with probability 1/2). The rate at which producers switch roles is equal to the number of producers, $P$, times the rate at which individual producers switch roles, $\alpha IP/2$. Similarly, the right-hand side of (2.8) is the rate at which middlemen are becoming producers. This switch occurs when a middlemen meets a consumer and concludes a sale, reducing his inventory by one unit. Over an arbitrarily short interval, each middleman meets

[5] It is assumed that producers trade with each other even if they are indifferent between trading and not trading. This assumption eliminates the multiple equilibria that would otherwise occur when $p_M = \gamma$.

a consumer with probability $\alpha(1-N)$, and having met a consumer, completes a sale with probability $1-F(q)^2$. The rate at which middlemen become producers is equal to the number of middlemen, $M$, times the rate at which individual middlemen conclude sales.

It is assumed that no supplier can be forced to participate in the market economy. A non-participant has an EPDU of 0, and a supplier can switch from being a non-participant to being a producer simply by producing a unit of goods. Thus, an equilibrium in which every supplier is willing to participate is characterized by:

$$V \geq \gamma \tag{2.9}$$

It is assumed that suppliers participate if they are indifferent between participating and not participating.

## 3. Equilibrium

In any equilibrium, producers and middlemen set prices optimally, consumers optimally choose between buying and rejecting the goods offered to them, and the $N$ suppliers are active and optimally choose their roles. There are two kinds of equilibria: one in which some suppliers choose to be middlemen, and one in which no supplier makes this choice.

**Definition 1.** *An* ***equilibrium with middlemen*** *consists of an allocation of suppliers $(P, M)$, a set of expected present discounted utilities $(U, V, W)$, and a set of prices $(p_C, p_M, q)$ such that (2.1)–(2.9) are satisfied, $P$ and $M$ are strictly positive, and the prices are between $\gamma$ and 1.*

**Definition 2.** *An* ***equilibrium without middlemen*** *consists of a set of expected present discounted utilities $(U, V, W)$ and a set of prices $(p_C, p_M, q)$ such that (2.1)–(2.9) are satisfied, $P = N$, and the prices are between $\gamma$ and 1.*

The nature of these equilibria is described by the following propositions. The proofs are contained in the appendix.

Equilibrium does not exist for all parameter values, and the two types of equilibria do not co-exist.

**Proposition 1.** *Let $N$, $\alpha$ and $r$ be given. An equilibrium with middlemen exists if and only if:*

$$\alpha(1-N)\eta_2(\gamma) - r\gamma \geq \alpha(1-N)\eta_1(\gamma)$$

*Equilibrium with middlemen, if it exists, is not necessarily unique. An equilibrium without middlemen exists if and only if:*

$$\alpha(1-N)\eta_2(\gamma) - r\gamma < \alpha(1-N)\eta_1(\gamma)$$

$$\alpha(1-N)\eta_1(\gamma) - r\gamma \geq 0$$

*If an equilibrium without middlemen exists, it is unique.*

These conditions have immediate interpretations. A supplier who chooses to produce incurs a cost of $\gamma$ units of the numeraire good. This one-time cost is equivalent to a flow cost of $r\gamma$. The supplier is then able to offer a trade to any consumer he meets. The flow expected benefit associated with this trading opportunity is $\alpha(1-N)\eta_1(\gamma)$.[6] Thus, the condition:

$$\alpha(1-N)\eta_1(\gamma) - r\gamma \geq 0 \tag{3.1}$$

states that the benefit of production is at least as great as the cost—that trading with consumers is a worthwhile activity. If this condition holds, the following condition might also hold:[7]

$$\alpha(1-N)\eta_2(\gamma) - r\gamma \geq \alpha(1-N)\eta_1(\gamma) \tag{3.2}$$

Acquiring a second unit of goods involves a flow cost of $r\gamma$ but raises the flow benefit from $\alpha(1-N)\eta_1(\gamma)$ to $\alpha(1-N)\eta_2(\gamma)$. Equation (3.2) states that the benefit is at least as great as the cost, so that suppliers are willing to take on the middleman's role if the benefit is at least as great as the cost.

These inequalities imply that there are two critical values of $\alpha(1-N)/r$. If the first critical value is reached, suppliers are willing to produce. If the second critical value is reached, they are also willing to act as middlemen. The conditions that give rise to these activities are low discount rates, frequent meetings, and a large ratio of consumers to suppliers.

The middleman's larger inventory of goods raises the probability of making a sale at any given price, inducing him to set his price higher than the producer's price. However, the middleman's price might be set so high that he sells goods less quickly than a producer.

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[6] Note that this expression assumes that any unit of goods sold will be immediately replaced, so that the cost of engaging in trade is, as claimed, the start-up cost $\gamma$.

[7] The former condition must hold when the latter condition holds because $\eta_2(\gamma) < 2(\widetilde{p} - \gamma)(1 - F(\widetilde{p}) < 2\eta_1(\gamma)$ where $\widetilde{p} \equiv \pi_2(\gamma)$.

**Proposition 2.** *In an equilibrium with middlemen,*

(a) *Middlemen charge consumers a higher price than do producers. The rate at which they sell goods to consumers can be either greater or smaller than the rate at which producers sell goods to consumers.*

(b) *If the equilibrium is unique, an increase in $\alpha$ or a decrease in $r$ will increase the number of middlemen and raise the price that they charge.*

## 4. Middlemen and Welfare

The effects that middlemen have upon welfare can be discovered by comparing the behaviour of an economy when some suppliers choose to be middlemen with the behaviour of the *same economy* when every supplier is a middleman.

The requirement that the two economies be identical in every respect except for the presence of middlemen is imposed so that the change in welfare can be unambiguously ascribed to their presence. It precludes a direct comparison between equilibrium with middlemen and equilibrium without middlemen, because Proposition 1 shows that there are no sets of parameters under which both types of equilibrium exist. Instead, equilibrium with middlemen will be compared to a reference equilibrium (called **producer equilibrium**) in which suppliers do not have the option of becoming middlemen. Producer equilibrium has all of the characteristics of equilibrium without middlemen, except that it exists whenever (3.1) is satisfied, and hence exists whenever equilibrium with middlemen exists.

### 4.1. Production Rate and Match Quality

Middlemen affect the economy in two ways: they change the rate at which heterogeneous goods are produced, and they change the manner in which these goods are allocated to consumers. These factors are measurable. The rate at which heterogeneous goods are produced is:[8]

$$\rho \equiv \alpha(1-N)\{P(1-F(p_C)) + M(1-F(q))^2\}$$

[8] The expression $\alpha\{\cdot\}$ is the probability with which a single consumer purchases a unit of the heterogeneous good, so $\rho$ is the rate at which goods are purchased. In equilibrium the rate of production is equal to the rate of sale since producers who sell to consumers replace immediately and (2.8) shows that the rate at which middlemen sell to consumers is equal to the rate at which producers sell to each other.

and the average consumption value of the goods sold is:[9]

$$\mu \equiv \alpha(1-N)\left[P\int_{p_C}^{1}\theta f(\theta)d\theta + 2M\int_{q}^{1}\theta f(\theta)F(\theta)d\theta\right] \div \rho$$

The production rate $\rho$ and match quality $\mu$ determine the welfare of the people living in the economy. Specifically, if $Z$ is the expected present discounted value of the consumer surplus, the flow surplus $rZ$ is:

$$rZ = \rho(\mu - \gamma)$$

It can readily be shown that:

$$Z = (1-N)W + MU + PV$$

That is, the agents in the economy are competing for shares of $Z$.

Producer equilibrium and equilibrium with middlemen differ in the equilibrium values of $\rho$ and $\mu$:

(a) Equilibrium requires units of the heterogeneous good to be produced and sold at the same rate. It follows that an economy with middlemen will produce goods at a faster rate than a producer economy if and only if middlemen sell goods more quickly than producers. That is, $\rho$ rises with the number of middlemen if and only if $F(q)^2$ is less than $F(p_C)$. Proposition 2 states that this condition might hold, but will not necessarily hold. The relative sizes of these probabilities depends upon the precise form of the distribution function, so the effect of middlemen on the rate of production is uncertain.

(b) Consumers who meet middlemen will, on average, buy more valuable goods than consumers who meet producers. There are two reasons for this outcome. First, they face higher prices and therefore have higher reservation values. Second, they have a choice of two goods rather than one, and choose the better of the two if both are acceptable. It follows that $\mu$ is greater in an equilibrium with middlemen than in a producer equilibrium.

[9] The numerator is the expected consumption value of the goods sold over some arbitrarily short interval, and the denominator is the quantity of goods sold over that interval. The ratio therefore represents average value.

| | $\alpha = 4.5$ | | $\alpha = 45$ | |
|---|---|---|---|---|
| | middlemen | producer | middlemen | producer |
| $\Delta$ | 0.255 | | 0.308 | |
| $P$ | 0.318 | 0.50 | 0.313 | 0.5 |
| $\rho$ | 0.513 | 0.45 | 5.026 | 4.5 |
| $\mu$ | 0.820 | 0.80 | 0.838 | 0.8 |
| $Z$ | 0.353 | 0.30 | 3.483 | 3.0 |
| $Y$ | 0.137 | 0.10 | 0.137 | 0.1 |
| $Z - Y$ | 0.217 | 0.20 | 3.346 | 2.9 |

Table 4.1: Comparing Equilibrium with Middlemen to Producer Equilibrium

If middlemen raise the rate of production, their presence unambiguously raises surplus. Otherwise, the impact of middlemen upon $Z$ depends upon the relative sizes of the changes in $\mu$ and $\rho$.

We examined a number of equilibria in which valuations were uniformly distributed, and were unable to find an instance in which $\mu$ and $\rho$ moved in opposite directions. Two of these equilibria are described in Table 4.1. In both sets, $N$ is equal to 0.5, $\gamma$ is equal to 0.2, and $r$ is equal to 0.9. The first set of calculations assumes that $\alpha$ is equal to 4.5, and the second set assumes that $\alpha$ is equal to 45.

### 4.2. The Cost of Delay

The variable $Z$ measures the present discounted value of surplus given that the economy begins in a particular equilibrium and remains there forever. It can be used to compare steady states, but it cannot be used to measure the welfare effects of moving from one equilibrium to another. That movement involves a transition period during which suppliers adopt new roles, prices adjust, and inventories are built up or run down. The welfare effects of these transitions cannot be accurately calculated because the dynamics of the model have not been completely specified.

Even in the absence of true dynamics, one might suspect that comparing $Z$ across economies would overstate the welfare gains generated by middlemen. Steady states with more middlemen also have higher levels of inventories, and these inventories must have been accumulated along the path to the steady state. Inventory accumulation requires goods to be produced but not sold, implying a reduction in consumer surplus. This loss might be amplified or offset by other elements of the dynamics, but it is surely important.

The size of this loss can be estimated by considering "test-tube economies" which begin in a steady state and stay there. In each steady state, some suppliers are initially producers and some are initially middlemen. Each producer is equipped with one unit of the heterogeneous good, and each middlemen is equipped with two. The cost of equipping the suppliers with their initial inventories is $Y$, where:

$$Y = \gamma(P + 2M)$$

This is only a set-up cost: once the economy has begun to operate, the agents will be responsible for restoring their inventories in the usual fashion. The net benefits associated with any given equilibrium are therefore equal to the present discounted value of the consumer surplus generated by that equilibrium, $Z$, less the set-up cost of the equilibrium, $Y$.

The flow of net benefits can be written as:

$$r(Z - Y) = \rho(\mu - \gamma) - rY$$

The last term is the flow cost of carrying inventories. It represents the time preference losses that arise because there is a delay between the production of goods and their ultimate consumption. Thus, this equation divides flow net benefits into the three components discussed in the introduction: rate of production $\rho$, match quality $\mu$, and the cost of delay in the trading process $rY$. The examples in Table 4.1 show that *each* of these components is higher when middlemen are present. This result is interesting because the efficiency gains generated by middlemen in the Rubinstein and Wolinsky model result from the middleman's ability to reduce the cost of delay.[10] In our examples, this result is reversed: there are efficiency gains even though middlemen, by holding larger inventories, increase the cost of delay.

## 5. Summary

A general equilibrium model in which some agents choose to become middlemen has been described. These agents ameliorate market frictions by offering consumers a choice of goods. Consumers typically find that the best of the goods

[10] In the Rubinstein and Wolinsky model, the costs of delay are reduced by middlemen only if the middlemen are more patient or match more frequently. Neither of these possibilities is allowed in our model.

offered by a middleman is more valuable to them than the good offered by a producer—that is, that the expected gains from trade between a consumer and a middleman are greater than those between a consumer and a producer. The middleman charges a higher price than does the producer, and buys from producers more cheaply than do consumers. The gains from mediation can be measured along three dimensions: rate of production, time preference losses generated by the matching process, and the quality of the match between consumers' preferences and the goods they ultimately consume.

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### Appendix

**Definition 3.** *Let $N$ and $A \equiv \alpha/r$ be given. A pair $(P, \Delta)$ is a* ***reduced solution*** *if it satisfies the equations:*

$$(P^2/2)I = (N-P)(1-N)\left[1 - F\left(\pi_2(\Delta)\right)^2\right] \tag{A1}$$

$$\Delta = A(1-N)[\eta_2(\Delta) - \eta_1(\gamma)] - (AP/2)\max[0, \Delta - \gamma] \qquad \textit{(A2)}$$

*and the inequalities:*

$$0 \leq P \leq N$$

$$0 \leq \Delta \leq 1$$

**Lemma 1.** *Let $N$ and $A \equiv \alpha/r$ be given, and let $\Delta^* < 1$ be the unique value of $\Delta$ satisfying the condition:*

$$A(1-N)[\eta_2(\Delta) - \eta_1(\gamma)] = \Delta$$

*Also, let $\widetilde{P}$ be the root of the quadratic equation:*

$$P^2/2 = (N-P)(1-N)\left[1 - F\left(\pi_2(\gamma)\right)^2\right]$$

*that lies between 0 and $N$. If $\Delta^* > \gamma$, there exists an odd number of reduced solutions, and each solution satisfies the inequalities $0 < P < \widetilde{P}$ and $\gamma < \Delta < \Delta^*$. If $\Delta^* = \gamma$, the only reduced solution is $(\widetilde{P}, \gamma)$. If $\Delta^* < \gamma$, the only reduced solution is $(N, \Delta^*)$.*

**Proof of Lemma 1.** Consider the cases $\Delta^* > \gamma$ and $\Delta^* \leq \gamma$ in turn.

(a) If $\Delta^*$ is greater than $\gamma$, the graph of (A1) in the $(P, \Delta)$ positive quadrant is continuous and downward sloping. By the definition of $\widetilde{P}$, one of its endpoints is $(\widetilde{P}, \gamma)$. Since $\lim_{\Delta \to 1} \pi_2(\Delta) = 1$, the other endpoint is (0,1). The graph of (A2) is also continuous and downward sloping. By the definition of $\Delta^*$, one of its endpoints is $(0, \Delta^*)$. To find the behaviour of the graph as $\Delta$ approaches $\gamma$, write (A2) as:

$$(AP/2)\max[0, \Delta - \gamma] = A(1-N)[\eta_2(\Delta) - \eta_1(\gamma)] - \Delta$$

Since the right-hand side is bounded away from zero when $\gamma \leq \Delta < \Delta^*$, this equation can only be satisfied if $P$ approaches infinity as $\Delta$ approaches $\gamma$, that is, if (A2) has an asymptote at $\Delta = \gamma$. To summarize: (i) both graphs are continuous and downward sloping in the $(P, \Delta)$ positive quadrant, (ii) the graph of (A1) lies above the graph of (A2) when $P$ is equal to 0, and (iii) the graph of (A1) lies below the graph of (A2) when $P$ is equal to $\widetilde{P}$. It immediately follows that the graphs must intersect an odd number of times over the domain $(0, \widetilde{P})$. Each point of intersection is a reduced solution.

(b) Now suppose that $\Delta^*$ is not greater than $\gamma$. Equation (A2) is only satisfied when $\Delta$ is equal to $\Delta^*$. Substituting this value into (A1) yields $P$. If $\Delta = \Delta^* = \gamma$, so that $I = 1$, $P$ is equal to $\widetilde{P}$. If $\Delta = \Delta^* < \gamma$, so that $I = 0$, $P$ is equal to $N$. ■

**Proof of Proposition 1:** Each equilibrium can be determined by block recursion. Equation (A2) is obtained by combining (2.4) and (2.6), and equation (A1) is an equilibrium condition, so each reduced solution gives a pair $(P, \Delta)$ from which an equilibrium can be developed. The prices are then determined by (2.3), (2.5) and (2.7); and the expected present discounted utilities are determined by (2.2), (2.4) and (2.6). Now consider the two kinds of equilibria:

(a) There are active middlemen if $P$ is less than $N$, which requires $\Delta^* \geq \gamma$, or equivalently,

$$A(1-N)[\eta_2(\gamma) - \eta_1(\gamma)] \geq \gamma$$

But $\eta_2(\gamma) < 2\left(\pi_2(\gamma) - \gamma\right)\left(1 - F(\pi_2(\gamma))\right) < 2\eta_1(\gamma)$, so this inequality implies that:

$$V = A(1-N)\eta_1(\gamma) > \gamma$$

as required by (2.9).

(b) There are no active middlemen if $P$ is equal to $N$, which requires:

$$A(1-N)[\eta_2(\gamma) - \eta_1(\gamma)] < \gamma$$

The inequality (2.9) need not then be satisfied, so an equilibrium with middlemen exists only if the inequality:

$$A(1-N)\eta_1(\gamma) \geq \gamma$$

is satisfied. ■

**Proof of Proposition 2:** (a) The price charged by producers, $p_C$, is the price $p$ that satisfies the first-order condition:

$$1 - F(p_C) = (p_C - \gamma)f(p_C)$$

implying:

$$\left(1 - F(p_C)\right)\left(1 + F(p_C)\right) = (p_C - \gamma)f(p_C)\left(1 + F(p_C)\right) > 2(p_C - \gamma)f(p_C)F(p_C)$$

The price charged by a middleman takes its smallest value when $\Delta$ is equal to $\gamma$. This minimum price, $\underline{q}$, is the value of $p$ that satisfies the first-order condition:

$$\left(1 - F(p)\right)\left(1 + F(p)\right) = 2(p - \gamma)f(p)F(p)$$

The second-order conditions for the middleman's optimization problem imply that the value of $p$ satisfying this condition exceeds $p_C$. The probability that a producer sells a unit of goods to a consumer is $1 - F(p_C)$ and the probability that a middleman sells a unit of goods is bounded above by $1 - F(\underline{q})^2$. These probabilities cannot be compared without placing strong restrictions on the form of the probability distribution. (b) The equilibrium is unique when the graphs of (A1) and (A2) cross only once. A change in the value of $A$ shifts only one curve, generating unambiguous comparative statics. ■

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